# Finding Collision Free Path with Probabilistic Roadmaps: The Bounding Volume Expansion*

Han-Ul Yoon

Department of Electrical and Computer Engineering
University of Illinois at Urbana-Champaign
1514 Beckman Institute, 405 N. Mathews Avenue, Urbana, Illinois 61801
hyoon24@uiuc.edu

**Abstract.** The efficiency of Probabilistic Roadmaps (PRM) depends on the number of samples in free configuration space and computation time to connect such samples. Key idea of this approach is to generate samples with bounding volume expansion and connect free samples based on geometric characteristics so that straight–line local planner does not do collision check while it connects a sample to another. This paper presents a geometric analysis in configuration space with bounding volume expansion and a sampling scheme to generate sample densely enough. Experiments show that 5–links robot finds a collision–free path in the environment where some obstacles are placed.

## 1 Introduction

Probabilistic Roadmaps (PRM) are an effective way to build collision-free path for the robots of which degree of freedom is high. PRM planner generates samples (randomly, uniformly, etc) in a robot configuration space and retain samples in collision–free space. Each sample becomes a node of graph structure. It also connects a node to its neighbor with collision check, which is done by local planner (simply, straight–line planner). Finally, it simplifies a path planning problem to typical graph search [1][2].

It has been proposed by Hsu, Sánchez-Ante, Cheng, and Latombe to dilate the free space for sampling narrow passages in PRM planning [3]. This method dilates free configuration space by thinning the geometric shapes of robots or obstacles and has been proved as efficient way in both running time and memory usage. Similar method also has been presented by Saha and Latombe – small step retracion method – to find a narrow passage [4]. The common key factor of these two method was to make robots and obstacles shrunken to obtain wide enough free space in configuration.

This paper presents a new idea of finding collision free path by the *bounding volume expansion*, which fattens a robot model, to guarantee small margin of free space with respect to a sample in free configuration. The paper also proposes a

* This research was the course project of ECE550 in spring 2007, University of Illinois at Urbana-Champaign.

sampling scheme by using *conditional probability* with respect to each angle range in configuration space to generate densely enough samples around a neighbor of a free sample. In the experiment, 5–links robot tries to find a collision-free path with PRM, however, no collision check process is performed by a local planner while it connects a sample to its neighbor.

The organization of this paper is as follows. In section 2, the expansion of bounding volume and geometric analysis are introduced. In section 3, a sampling scheme with a conditional probability is presented. In section 4, the paper shows that experimental results to find a free path with 5-links robot. In section 5, conclusions and further works are presented.

## 2 Bounding Volume Expansion and Geometric Analysis

In computer graphics and computational geometry, a *bounding volume* for a set of objects is a closed volume that completely contains the union of the objects in the set [5]. In collision detection between two objects, we can determine whether these objects collide each other by looking the spatial relationships of two bounding volumes. If two bounding volumes do not intersect, then the contained objects cannot collide, either.

A good way to understand "*How to apply the bounding volume expansion to sample connection with a local planner?*" is to consider the following problem. Three airbourn rangers were waiting for a jump inside C–130 (a cargo airplane) to operate a secret mission. Unfortunately, the headquarter reported that their landing zone was being suspected that the enemy had installed a thousand of landmines. Therefore, the captain set a plan as follows: First, they threw five titanium disks, e.g., 10-feet diameter, down to the landing zone. Next, they would be able to land on the disks. After throwing the disks and several sounds of explosion, they could see that the disks were placed on the landing zone. Moreover, some of the disks were folded partially together. Figure 1 depicts an illustrative example of this situation.

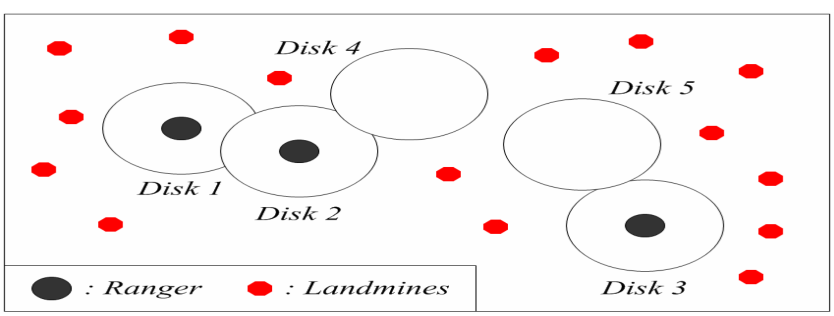


**Fig. 1.** Airbourn rangers landed on the disks in landmine field

Interestingly, throwing a titanium disk corresponds to generating a sample with the bounding volume expansion. A sample, which is generated with the bounding volume expansion, has a marginal free space around itself as well a ranger could take free move on the disk without stepping on landmines. Also, on the partially folded two disks, e.g., *Disk 1* and *Disk 2* in Fig. 1, each ranger can move toward another disk and do not need to turn on a landmine detector, even if they do not stand on the ground. This is one of key operations in PRM planning with bounding volume expansion.

Figure 2 shows that two-links planar arm robot and its configuration space. A solid line area presents a obstacle in configuration space, and a dashed line area shows that it is also expanded as we apply the bounding volume expansion. As remind airbourn ranger example, move inside the disk corresponds to rotate the angle, $\theta_1$ and $\theta_2$ within the volume expanded area. Let's assume that each link has the same length $l$. Now, when link1 move toward $+\Delta\theta_1$, link2 can be placed inside the expanded area by rotation $\theta_2 - \Delta\theta_1$ (and vice versa). If we did set a radius of $r$ volume expansion, then $\Delta\theta_1$ can be determined by $2^{nd}$ cosine law. Thus, $\Delta\theta_1 \simeq cos^{-1}(\frac{2l^2-r^2}{2l^2})$. This scenario is presented in Figure 3.

We assumed that each link of the robot has the same length. Now, let's denote that $\theta_i^k$ the $i^{th}$ angle of the robot in the $k^{th}$ sample $q_k$. With this notation, let's assume that there are two sample points, which are $q_1(\theta_1^1, \theta_2^1)$ and $q_2(\theta_1^2, \theta_2^2)$, so $q_1(\theta_1^1 \pm \Delta\theta_1^1, \theta_2^1 \mp \Delta\theta_1^1)$ and $q_2(\theta_1^2 \pm \Delta\theta_1^2, \theta_2^2 \mp \Delta\theta_1^2)$ are correspoding free space margin. In Figure 4, we can verify that if a distance between $q_1$ and $q_2$, $dist(q_1, q_2)$, is shorter than $dist(q_1, q_1') + dist(q_2, q_2')$, then their is at least one point that $\bar{q_1} - \bar{q_1}'$ intersects $\bar{q_2} - \bar{q_2}'$ or vice versa. Figure 4(a) shows an illustrative example. Again, let's remind that there were the partially folded disks in the airbourn ranger example. The point in the intersection of $\bar{q_1} - \bar{q_1}'$ and $\bar{q_2} - \bar{q_2}'$ corresponds to the part that two disks meet together. Therefore, if we denote this intersect point with $p_{12}$, we know that $q_1 \rightarrow p_{12} \rightarrow q_2$ can be connected without collision check process.

Figure 4(b) shows 5–links robot for this project. In this case, we know that if $dist(q_1, q_2) \leq dist(q_1, q_1') + dist(q_2, q_2')$, then we can directly connect $q_1 \rightarrow q_2$

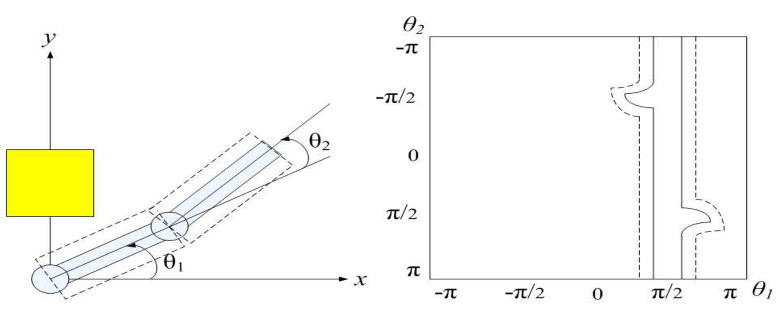


**Fig. 2.** Two-links planar arm robot and its configuration spaces.

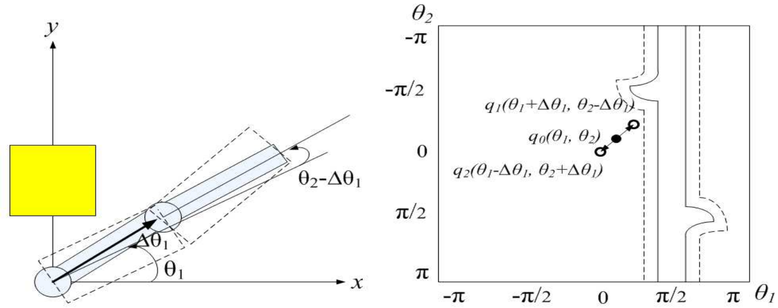


**Fig. 3.** Link1 rotates $\Delta\theta_1$, link2 rotates $\theta_2 - \Delta\theta_1$. These rotations allow the robot being placed the inside of expanded area.

via $p_{12}$, where $q_1(\theta_1^1, \alpha_1^1, \theta_2^1, \alpha_2^1)$, $q_2(\theta_1^2, \alpha_1^2, \theta_2^2, \alpha_2^2)$, $q_1(\theta_1^1 \pm \Delta\theta_1^1, \alpha_1^1 \pm \Delta\alpha_1^1, \theta_2^1 \mp \Delta\theta_1^1, \alpha_2^1 \mp \Delta\alpha_1^1)$, and $q_2(\theta_1^2 \pm \Delta\theta_1^2, \alpha_1^2 \pm \Delta\alpha_1^2, \theta_2^2 \mp \Delta\theta_1^2, \alpha_2^2 \mp \Delta\alpha_2^2)$ respectively.

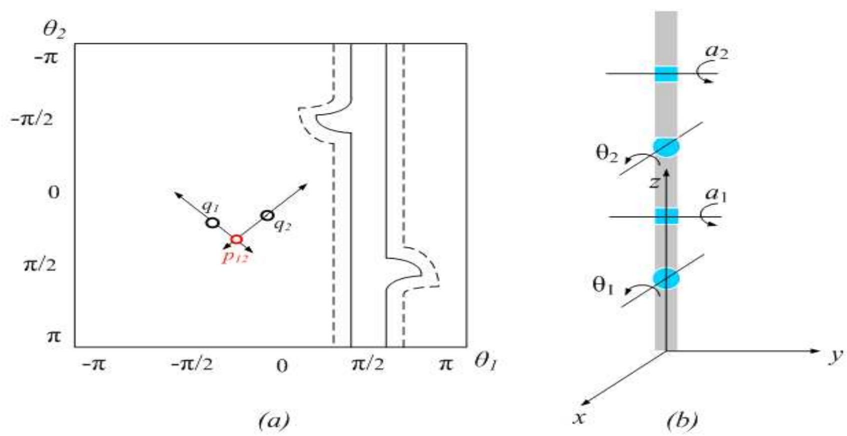


**Fig. 4.** (a) Direct sample connection via intersect point, (b) 5–links robot with 2–$\theta$ and 2–$\alpha$ rotations.

## 3 Sample Generation Scheme

In the former section, we verified the condition that two samples in configuration space can be connected directly via an intersection point. However, it turns out there are some problems with this method. Since we can connect two samples without collision checking if and only if $dist(q_1, q_2) \leq dist(q_1, q'_1) + dist(q_2, q'_2)$.

One solution is to generate sample uniformly with a small interval. However, if the size of interval is 5 in degree, for example, the number of total sample for 5-links robot (since base frame is fixed to the ground, the DOF of this robot is 4) will be $(365/5)^4 = 75^4 \simeq 3.2 \times 10^7$. It is obviously wasteful computation to check the connectivity of all samples in this case.

Another solution can be to generate sample based on the conditional probabilities of collision samples, given the range of angle $a \leq \theta_{ij} < b$ and $c \leq \alpha_{ij} < d$, where $i$ is the angle of $i^{th}$ link and $j$ is the $j^{th}$ range of this angle. First, we generate specific amount of samples, e.g., a hundred, then check the probabilities of collision samples $P(collision|a \leq \theta_{ij} < b)$ and $P(collision|c \leq \alpha_{ij} < d)$. The coefficients – $a$, $b$, $c$, and $d$ – are the start angle and the end angle of each range. For example, if we determine the angle range of the $1^{st}$ link to be 36 in degree, then the given conditions will be $0 \leq \theta_{11} < 36$, $36 \leq \theta_{12} < 72$, $72 \leq \theta_{13} < 108$, and so on. After generating enough samples, calculate $P(collision|\theta - range_j)$ and $P(collision|\alpha - range_j)$, where $j = 1, 2, 3, \cdots, j$.

These conditional probabilities give us very useful information. First, it give us the approximate distribution of obstacles. Second, if we regard these $j$–ranges of each $\theta_i$ and $\alpha_i$ as the attribute values that describe the instance of classification problem, these value can be used for the naïve Bayes classifier to classify a sample dropped in free configuration space or not. As we know, the naïve Bayes classifier has above 95 percent average when it classifies the known instance (the performance will decrease when an instance involves unknown attributes) [6][7]. However, let's put aside the naïve Bayes classifier application for now, because we rather use these conditional probabilities to set the *Roulette wheel* in Fig. 5.

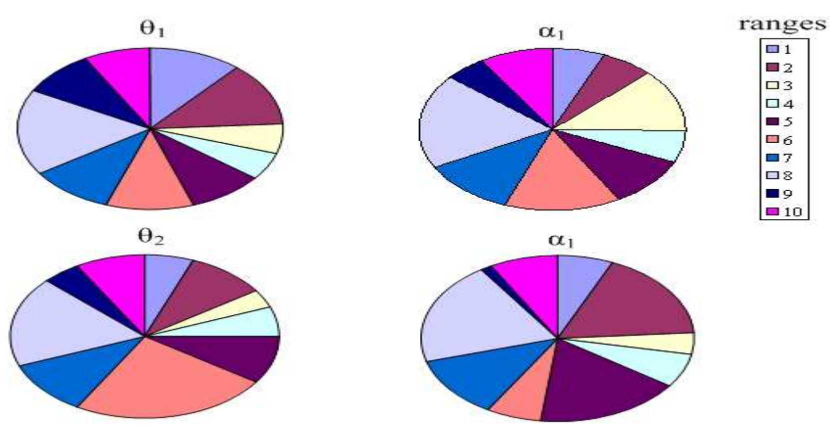


**Fig. 5.** 4-Roulette wheels with assigning the conditional probability with respect to each range

Figure 5 shows the Roulette wheel with assigning $P(collision|range_j)$ with respect to $\theta_i$ and $\alpha_i$. It means that the range where has the higher probability will get more chance to generate a sample in the next time. For example, if the range 5 is selected by the roulette wheel of $\theta_1$, then the next sample will be generated between 144 and 180 in degree. By this method, we can generate sample more densely inside a free configuration space. Even though this method somewhat guarantees that a free configuration will be sampled densely, their is still posibilities that a sample cannot find its neighbor which is connectible directly. Unfortunately, we should use normal straight–line planner with collision check process for this case. Finally, the algorithm to build a roadmaps with the bounding volume expansion is depicted in Table 1.

**Table 1.** Probabilistic roadmaps with bounding volume expansion

1: For each *vertex* and *edge* in the graph $G$
  Initialize $v_i$ and $e_{ij}$
2: Generate samples $s_i$
3: Do until the roadmap building is completed
  if $s_i \in C_{free}$ then
    • Add $v_i$ to $G$
    • $i \leftarrow i + 1$
4:     for each $s_j \in$ NEIGHBORHOOD($s_i$, $G$)
        if $dist(s_i, s_j) \leq dist(s_i, s_i') + dist(s_j, s_j')$
          • CONNECT($s_i, s_j$) directly via $p_{ij}$
        else
          • CONNECT($s_i, s_j$) with collision check
      Add $s_j$ to $e_{ij}$
5: Goto 3

## 4 Experiments with Five–Links Robot

This experiments were performed by using *Crystal Space*, 3D game engine, with five-links robot. The bounding volume expansion and introduced sampling scheme were applied, when generate samples in configuration.

**Sample Generation** Table 2 shows that $\Delta\theta_i$ and the number of generated free samples out of 50 samples when we apply $r$ bounding volume expansion. First, we define *type1* and *type2* samples as follows: a) *type1* is a sample that it is in collision free space, and is also collision free under the bounding volume expansion; b) *type2* is a sample that it is in collision free space, but is not collision free under the bounding volume expansion. Figure 6 depicts that the number of *type2* samples increase as a function of $r$.

**Table 2.** The number of *type*1 samples, *type*2 samples, and $\Delta\theta$ with respect to different $r$ bound (out of 50 samples, 4 landmines).

| $r$ bound | $\Delta\theta$ | free samples | *type*1 | *type*2 | Remark |
|---|---|---|---|---|---|
| 0 | 0 | 30 | 30 | 0 | no expansion |
| 0.0 | 0 | 30 | 30 | 0 | |
| 0.2 | 11 | 30 | 26 | 4 | |
| 0.4 | 23 | 30 | 25 | 5 | |
| 0.6 | 34 | 30 | 19 | 11 | |
| 0.8 | 47 | 30 | 13 | 17 | |
| 1.0 | 60 | 30 | 0 | 30 | full expansion |

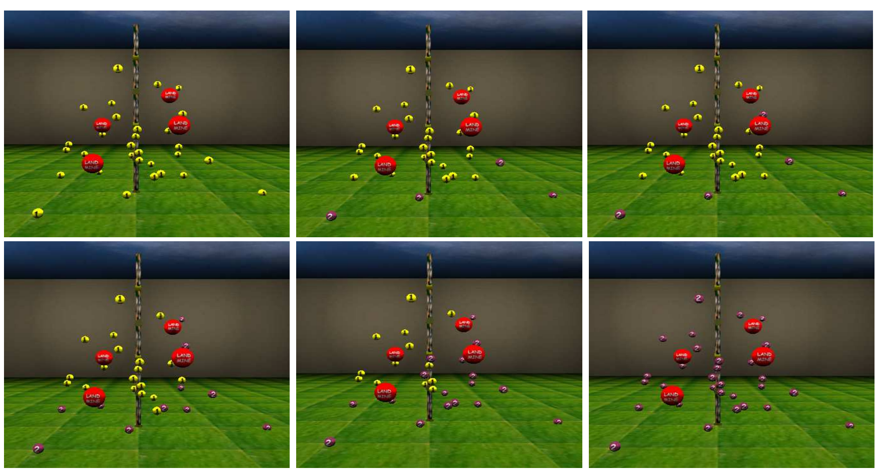

**Fig. 6.** The bigger $r$ generates, the more *type*2 samples (the purple small circles in the figure).

Figure 7 shows that the sample generation when we apply *RouletteWheel* generation with the given conditional probabilities of each region of $\theta_i$. Compared with (a1) to (a2), and (b1) to (b2), We can see that the samples are placed more densely in free space. In this case, we divided each angles to 4 regions, which are 0∼90, 90∼180, 180∼270, and 270∼360 respectively.

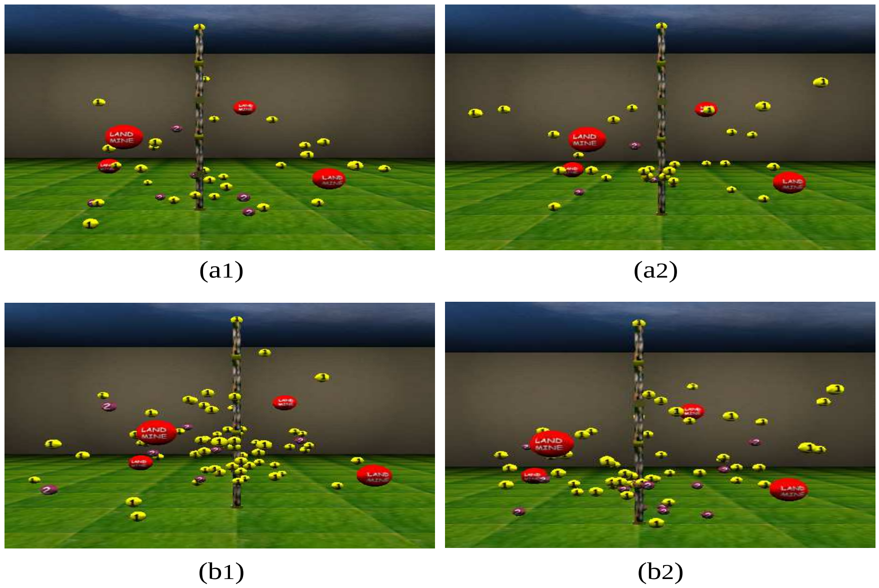


**Fig. 7.** Samples are generated more densely under the *RouletteWheel* generation.

**PRM planner with the bouding volume expansion.** Figure 8 shows that four successful runs of PRM under the bounding volume expansion. we did set up $r$ from 0.2 to 0.8 in 0.2 increment, and sample size 100 (it became doublesized to 200). As we also applied normal straight–line local planner, the most of trials we can found a goal. The algorithm, in Table 1, returned total number of directly connected samples after finishing each run. According to the result, we had 5 directly connected samples in this case. When we increased sample size to 200, the number of directly connected samples also increased to 11. However, the problem is that this method always doublesizes the number of nodes, because it has to put a new node, $p_{ij}$ whenever it finds two samples that their direct connection is plausible. Therefore, it took approximately more than a hour to connect the nodes in a graph when we applied this method with 1000 samples.

## 5 Conclusions and Further Works

This paper presented the probabilistic roadmaps with the bounding volume expansion. Five–links robots were used to verify that we can connect the samples directly if and only if $q_2$, $dist(q_1, q_2) \leq dist(q_1, q_1') + dist(q_2, q_2')$. Also, we introduced the sample generation scheme to generate samples densely enough by using a conditional probabilities of the range of each angle. The experimental results from the application of the bounding volume expansion were presented.

This method can be a new way to reduce a time complexity and innovate the performance of PRM. The advantage is that we can connect most of sam-

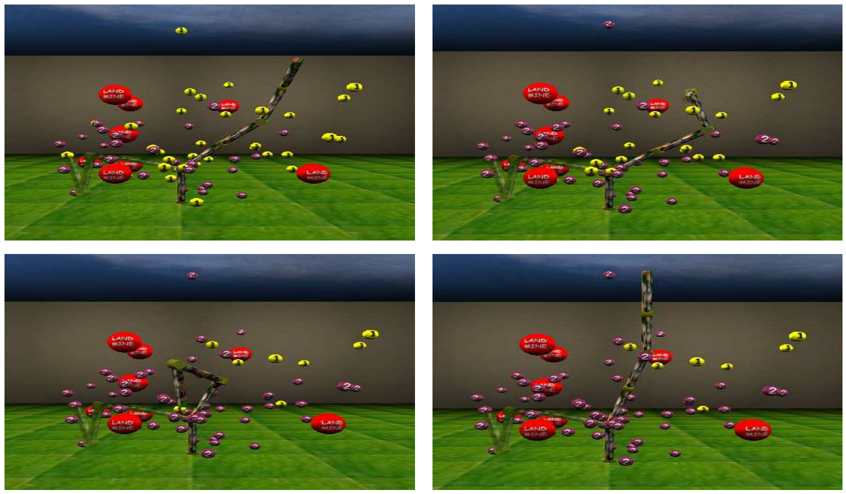

**Fig. 8.** Successful runs under the bounding volume expansion. We can also see that *type*2 samples increase as $r$ becomes from 0.2 to 0.8

ples without collision checking, when we generate sample closely to its neighbor. However, if we apply the bounding volume expansion, the probability of generating samples in narrow passage is extremely low. Therefore, we need to clarify the problem of accessing a narrow passages. Another problem is that two configurations are connected via the intersect point, $p_{ij}$, under the bounding volume expansion. We can know intuitively that adding up $p_{ij}$ doublesizes the number of node in roadmap approximately. Also, it is hard to find appropriate $p_{ij}$ when the maximum number of samples are small (e.g. less than 100). From the experiment, at least 200 hundred samples (approximately 400 when there are double sized) are required. Therefore, specially defined data structure should be developed to clear these problems.

For the future research, first, we need to apply this method to more complex robots, e.g., 7-links robot or more. Second, the robot simulation code desires to be improved so that the program routine has a succinct structure for the future application. Third, machine learning techniques should be applied so that robots can connect such a sample to its neighbor based on the hypothesis what they learnt. Finally, the total system needs to be refined to obtain better results.

## 6 Acknowledgement

I thank to Prof. *Seth Hutchinson* and T.A. *Sal Candido*. for inspiring me, encouraging me, and consoling me, in this course. Also, it was my honor to be with my outstanding classmates: *Aaron*, *Brad*, *Carlos*, and *Jaesik*. I do not know how I can express my great thanks to my wife, *Logie*. This semester was

miserable enough for us to tolerate, but keep believing me, and we are actually getting better.

## References


1. M. Spong, S. Hutchinson, and M. Vidyasagar, *Robot Modeling and Control*. John Wiley and Sons, 2006, pp.163–182.
2. H. Choset, K. Lynch, S. Hutchinson, G. Kantor, W. Burgard, L. Kavraki, and S. Thrun, *Principles of Robot Motion*, MIT press, 2005, pp.77–106.
3. D. Hsu, G. Sánchez-Ante, H.-l. Cheng, and J.-C. Latombe, "Multi-level free-space dilation for sampling narrow passages in PRM planning," *Proceeding in IEEE International Conference on Robotics and Automation*, May 15-19, 2006, pp.1255–1260.
4. M. Saha and J.-C. Latombe, "Finding narrow passages with probabilistic roadmaps: the small step retraction method," *IEEE/RSJ International Conference on Intelligent Robots and Systems*, 2-6 Aug. 2005. pp.622–627.
5. Wikipedia. (2007). Bounding Volume: Uses of bounding volumes. [Online]. Available: http://en.wikipedia.org/wiki/Bounding_volume.
6. T. Mitchell, *Machine Learning*, McGraw-Hill, 2006, pp.154–180.
7. S. Russell and P. Norvig, *Artificial Intelligence: A Mordern Approach 2nd Eds.*, Prentice Hall, 2003, pp.462–482.